# Identification of Inclined Ionospheric Layers Using Analysis of GPS Occultation Data 

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#### Abstract

The ionosphere and atmosphere may have significant impacts on the high-stable navigational signals of the Global Positioning System (GPS) in the communication link satellite to satellite. The classification of the different types of the ionospheric impact on the phase and amplitude of the GPS signals at altitudes of $\mathbf{4 0 - 9 0} \mathbf{~ k m}$ is introduced using the CHAllenging Minisatellite Payload (CHAMP) radio occultation (RO) data. An analytical model is elaborated for the description of the radio wave propagation in the stratified ionosphere and atmosphere. The propagation medium consists of sectors having the spherically symmetric distributions of refractivity. The newly developed model presents analytical expressions for the phase path and refractive attenuation of radio waves. The model explains significant amplitude and phase variations at altitudes of $\mathbf{4 0 - 9 0} \mathbf{~ k m}$ of the $R O$ ray perigee associated with the influence of the inclined ionospheric layers. An innovative eikonal acceleration technique is described and applied to the identification and location of the inclined ionospheric layers using the comparative analysis of the amplitude and phase variations of the RO signals.


Index Terms-Atmosphere, GPS, ionosphere gradient, LEO, radio occultation.

## I. Introduction

LIMB sounding radio occultation (RO) observations of the Earth's atmosphere and ionosphere have been recently carried out using Global Positioning System (GPS) radio signals at two frequencies $f_{1}=1575.42 \mathrm{MHz}$ and $f_{2}=$ 1227.60 MHz with a global coverage (e.g., [2], [7], [9], [10], [12], and [20]). When applied to the ionospheric investigations, the RO method can be compared with the global Earth- and space-based radio tomography (e.g., [3], [4], [6], [8], and [18]). The horizontal gradients in the ionosphere may introduce significant interference in the RO studies of the ionosphere and

[^0]

Fig. 1. Geometry of a radio ray path in the RO experiment.
atmosphere. The mechanism of the ionospheric interference may be revealed by analyzing the radio wave propagation through locally spherical symmetric media.

During a RO event, the radio ray linking a LEO GPS receiver to a GPS satellite transmitter (marked by points $L$ and $G$ in Fig. 1, respectively) gradually immerses into the ionosphere and atmosphere. The LEO receiver measures the GPS phase delays and amplitudes of the RO signal for subsequent retrieval of the vertical profiles of the physical parameters of the ionosphere and atmosphere. Suppose that multipath propagation is absent, the RO inversion technique is based on two implicit assumptions: 1) the tangent point $T$, where the refractivity gradient is perpendicular to the RO ray direction, coincides with the RO ray perigee (Fig. 1), and 2) there is no other such point on the trajectory GTL. Under these assumptions, despite a prolonged path $G T L$, a relatively small area, with its center at point $T$, introduces the main contribution to the amplitude and phase variations of the RO signals [19]. In this case, the amplitude and phase variations of the RO signals are functions of the ray perigee height $h(T)$ (Fig. 1) and the satellites' positions and velocities. These functions can be used to determine the height dependence of the bending angle $\xi(p(h))$ if precise orbital data are given [12]. Abel transform is then applied to obtain the vertical profiles of the electron density in the ionosphere and the refractivity in the atmosphere as functions of the ray perigee height $h$ [12]. From the amplitude, the vertical gradients of the electron density in the ionosphere and the refractivity in the atmosphere can be independently obtained [11], [19].

Usually, the ionospheric influence may be described as a relatively slow change in the phase path excess without noticeable variations in the amplitude of the RO signals. This change can be excluded by different methods of ionospheric correction [12], [17]. The strong amplitude and phase frequency dependent variation phenomenon in the RO signals are sometimes observed within the $40-90 \mathrm{~km}$ altitudes above the main part of the neutral atmosphere and below the E-layer of the ionosphere. This effect is analyzed in this paper.


Fig. 2. Amplitude and phase measurements of the CHAMP RO signals in both (left) quiet and (right) slightly disturbed ionosphere (events No. 0174 and 0023 ; February 24 and June 16, 2003). The LT of the event and the coordinates of the atmospheric tangent point $T$ are shown in the graphs. The $S_{4}$ index value is shown in the lower line of the insert.

The goals of this paper are the following: 1) to introduce different kinds of ionospheric impacts on the RO signals within the $40-90 \mathrm{~km}$ altitudes; 2) to present an analytical model for the refractive attenuation and phase path excess of the electromagnetic waves in locally spherical symmetric media; and 3) to demonstrate the possibility to identify contributions and to measure parameters of the inclined plasma layers by analyzing CHAMP experimental data. This paper is structured as follows. Section II introduces the different kinds of ionospheric impacts on the CHAMP RO signals. Section III presents an analytical model of wave propagation through the ionosphere and atmosphere of the Earth. Section IV gives an example of the ionospheric layer identification and the electron density retrieval. The conclusion is given in Section V.

## II. Types of Ionospheric Influence on the CHAMP RO Signal

The RO experiment aboard CHAMP was activated on February 11, 2001 [19]. The carrier phase at frequencies $f_{1}$ and $f_{2}$ and the signal-to-noise ratio at $f_{1}$ are recorded at a sampling rate of 50 Hz . Previously, the RO technology has been based mainly on analyzing the phase of the electromagnetic wave after propagating through the ionosphere and atmosphere [7]. The amplitude of the RO signals presents new potential and capability for the research and observations of the atmosphere and ionosphere [10], [11], [16]. The high sensitivity of the amplitude variations to ionospheric plasma layers can be used to describe different kinds of ionospheric impacts on the RO signals. For the CHAMP RO experiments, quiet ionospheric conditions have come to light in the form of small values of the $S_{4}$ index of the amplitude scintillations averaged at the altitudes $H$ in the $40-90 \mathrm{~km}$ interval

$$
\begin{equation*}
S_{4}=\left[\left\langle(I(t)-\langle I\rangle)^{2}\right\rangle /\langle I\rangle^{2}\right]^{1 / 2} \tag{1}
\end{equation*}
$$

where $I$ is the intensity of the RO signals and $\langle I\rangle$ is the average intensity of the RO signals. An example of the quiet ionospheric
conditions observed during the CHAMP RO experiments is shown in Fig. 2 (left panel). The amplitude curve 1 has low variations, with index $S_{4}=1.7 \%$. The phase path excesses $\Phi_{1}(t)$ and $\Phi_{2}(t)$ at $f_{1}$ and $f_{2}$ are shown by curves 2 and 3, respectively. Curve 4 corresponds to the combined phase $\Phi(t)$ from the following ionospheric correction formula [12]:

$$
\begin{equation*}
\Phi(t)=\left[f_{1}^{2} \Phi_{1}(t)-f_{2}^{2} \Phi_{2}(t)\right] /\left(f_{1}^{2}-f_{2}^{2}\right) . \tag{2}
\end{equation*}
$$

Examples of the significant variations of the phase and amplitude of the GPS RO signals are given in the following in the altitude intervals of the ray perigee $40-90 \mathrm{~km}$. These examples support the suggestion that there exist inclined ionospheric layers located along the RO ray trajectory.

In Fig. 1, the apparent displacement of the height of the ionospheric layer $\Delta h$ can be used in estimating its inclination $\delta$ with respect to the local horizontal direction [19]

$$
\begin{equation*}
\delta \approx(2 \Delta h / a)^{1 / 2} \tag{3}
\end{equation*}
$$

where $a$ is the Earth's radius (Fig. 1). One can estimate, using Fig. 1, the horizontal distance $d$ between the atmospheric and ionospheric tangent points if $\Delta h$ is known.

The isolated quasi-regular event with the influence of the sporadic E-layer is shown in Fig. 2 (right panel). The fine structures corresponding to an inclined sporadic E-layer are seen in the height $h(T)$ intervals of $45-70 \mathrm{~km}$. The phase and amplitude vertical distributions correspond to the inclined sporadic E-layers that are usually located at a height of about $93-103 \mathrm{~km}$ in the evening ionosphere. One can estimate the displacement of the ionospheric tangent point using (3). For $\Delta h=h^{\prime}-h \approx 50 \mathrm{~km}$, one can obtain $\delta \approx 6^{\circ}$ and $d \approx$ $(2 \Delta h a)^{1 / 2} \approx 700 \mathrm{~km}$. These values may correspond to the sporadic E-layer at a height of 98 km , declined by $6^{\circ}$ relative to the horizon and located at a distance of 700 km away from the atmospheric tangent point in the RO plane. An example of the event with high quasi-regular variations in the amplitude and phase of the CHAMP RO signal, with the magnitude of index $S_{4}=17 \%$, is shown in Fig. 3(a). The eikonals at two


Fig. 3. (a) Quasi-regular variations in the amplitude and phase values of the CHAMP RO signals for event No. 0093 (February 24, 2003). (b) Amplitude and phase values of the CHAMP RO signals due to the diffraction of the electromagnetic waves in the ionosphere (event No. 0027; February 24, 2003). Curve 1 corresponds to the amplitude variations, and curves 2 and 3 are related to the eikonal variations at $f_{1}$ and $f_{2}$, respectively.
frequencies change in the interval $6 \leq \Phi_{1,2} \leq 10 \mathrm{~m}$. These variations may be associated with layers in the electron density distribution. A strong ionospheric influence with the diffraction structures in the RO signals is shown in Fig. 3(b) at the heights of $50-52.5 \mathrm{~km}$. This case can be considered as a consequence of the diffraction of the electromagnetic waves on the sharp gradients of the electron density in an inclined sporadic E-layer. The examples of the noisy CHAMP events with significant amplitude variations $\left(S_{4}=10 \%\right)$ are shown in Fig. 4. These events can be classified as typical cases, which are relevant to noisy ionospheric contributions caused by ionospheric irregularities in the equatorial region near midnight at 22 h 31 m and 23 h 21 m of the local time (LT) [Fig. 4 (left and right panels, respectively)]. The phase variations indicate sharp changes in the total electron content, which may correspond to bubbles moving in the disturbed region of the ionosphere.

Therefore, five types of ionospheric influence on the RO signals can be established at altitudes of $40-90 \mathrm{~km}$. They are listed as follows

1) quiet events [type 1; Fig. 2 (left)];
2) isolated quasi-regular flashes in the amplitude and phase [type 2; Fig. 2 (right)];
3) regular events with quasi-periodical RO amplitude variations [type 3; Fig. 3(a)];
4) diffractive ionospheric events [type 4; Fig. 3(b)];
5) noisy events with highly incoherent ionospheric contributions to the amplitude of the RO signal (type 5; Fig. 4).

These types can be compared with the results of long-term (16 months) measurements of the signals propagating from a MARISAT satellite over the Indian Ocean at an elevation angle of $17.3^{\circ}$ [5]. Both C- and S-types of the ionospheric amplitude scintillations of the radio signals are identified. The C-type is similar to noisy variations without any significant regular or periodical structure in the amplitude changes of the transionospheric signals. The S-type consists of quasi-regular structures which can be associated with the influence of the bubbles or other types of disturbances in the ionospheric plasma [5]. The CHAMP RO amplitude variations can also be recognized as the C- and S-types of the amplitude scintillations previously classified in the communication INMARSAT link at the same frequency band near 1.5 GHz [5]. The noisy C-type amplitude variations in the CHAMP RO signals at altitudes of 30120 km are shown in Fig. 5 (left panel). Curves 1, 3, 4, and 5 describe the scintillations observed in the North and South Polar regions during the RO events of No. 0170, 0025, 0047, and 0196 (January 23, 2003). For events No. 0170, 0025, and 0196, the $S_{4}$ index was equal to $0.12,0.13$, and 0.21 (curves 1,3 , and 5 , respectively), which corresponds to slightly disturbed ionospheric conditions. Event No. 0047 corresponds to the weakly disturbed ionosphere, with the $S_{4}$ index being equal to 0.053 . Curves 2 and 6 (RO events No. 0019 and 0198; January 23, 2003) are relevant to the mid-latitudes with quiet (curve $2 ; S_{4}=$ 0.038 ) and slightly disturbed (curve $6 ; S_{4}=0.12$ ) ionosphere. The amplitude fluctuations during the RO event No. 0019 in the height interval of $h(T)=30-110 \mathrm{~km}$ (curve 2) were mainly caused by random receiver noise. The geographical


Fig. 4. Amplitude and phase values of the CHAMP RO signal for noisy events No. (left) 0069 and (right) 0014 (February 24, 2003) near the geomagnetic equator at local nighttime. Curve 4 (right panel) corresponds to the combined phase $\Phi(t)$ from formula (2).



Fig. 5. (Left) C-type noisy and (right) S-type quasi-regular amplitude scintillations of the CHAMP RO signals. The legends indicate the LT, the geographical coordinates, the magnitude of the $S_{4}$ index, and the event's number of the RO experiments.
position and LT of the noisy RO events correspond to the same parameters of the noisy amplitude scintillations previously observed in transionospheric communications (e.g., [21]). The quasi-regular S-type amplitude variations are shown in Fig. 5 (right panel). Curves 2 and 4 correspond to the CHAMP RO events No. 0137 and 0015 (January 23; mid-latitude nighttime ionosphere), respectively; curves 1,3 , and 5 relate to events

No. 0083, 0192, and 0076; and curve 6 corresponds to event No. 0205 on January 23, 2003 (equatorial daytime; south and north polar ionosphere). The inclined plasma layers in the E- or F-regions of the ionosphere might be the cause of the quasi-regular amplitude variations. Through comparisons of the data shown in Figs. 2-5, types 2, 3, and 4 of the RO amplitude variations can be recognized as the S-type amplitude


Fig. 6. Section of the ray tube by the RO plane containing transmitter $G_{1}$, receiver $L$, and centers of the local spherical symmetry $O_{1}, O_{2}$, and $O_{3}$. Lines $G_{1} L$ and $G_{1} L^{\prime}$ are the top and bottom boundaries of the ray tube, respectively. Points $B_{1}$ and $B_{2}$ are the boundaries between two parts of the ionosphere $G_{1} B_{1}$ and $B_{2} L$ and the atmosphere $B_{1} B_{2}$, and angles $\xi_{1}, \xi_{2}$, and $\xi_{3}$ are the bending angles corresponding to the three parts of the RO ray $G_{1} B_{1}, B_{1} B_{2}$, and $B_{2} L$.
scintillations observed in the transionospheric INMARSAT link by [5]. The noisy variations in the CHAMP RO signals correspond to the amplitude scintillations of the C-type. This coincidence in the types of CHAMP RO amplitude scintillations and the amplitude variations observed in the Earth-based experiments indicates the common ionospheric mechanisms of their origin.

## III. Analytical Model for the Phase Path Excess and Refractive Attenuation of the RO Signal

A model of a medium composed of several spherical sectors was previously applied to estimate the effects of horizontal gradients in the atmosphere (e.g., [1]). A new analytical model describing the changes of the phase path and refractive attenuation of the electromagnetic waves propagated in the ionosphere and atmosphere is introduced in this paper. Three centers of local spherical symmetry associated with a single RO ray path in the ionosphere and atmosphere are located at different points $O_{1}, O_{2}$, and $O_{3}$ corresponding to three parts of the ray trajectory $G_{1} B_{1} B_{2} L$ in three spherical sectors having boundaries at points $B_{1}$ and $B_{2}$, respectively: $G_{1} B_{1}$ (the ionosphere between transmitter $G_{1}$ and the atmosphere), $B_{1} B_{2}$ (atmospheric part), and $B_{2} L$ (the ionosphere between the receiver and the atmosphere; Fig. 6). Points $G_{1}, L, O_{1}, O_{2}$, and $O_{3}$ are assumed to belong to the plane $G_{1} B_{1} B_{2} L$. This assumption corresponds to the experimental RO data indicating a significant refraction effect in the plane of propagation. The central angles $\theta_{1}, \theta_{2}$, and $\theta_{3}$ between directions to transmitter $G_{1}$ and receiver $L$ have vertices located at points $O_{1}, O_{2}$, and $O_{3}$ (Fig. 6). The distances $G_{1} O_{1}, G_{1} O_{2}$, and $G_{1} O_{3}$ and $L O_{1}, L O_{2}$, and $L O_{3}$ are equal to $D_{1}, D_{2}$, and $D_{3}$ and $R_{1}(L)$, $R_{2}(L)$, and $R_{3}(L)$, respectively. Lines $G_{1} L$ and $G_{1} L^{\prime}$ are, accordingly, the sections of the top and bottom boundaries of a ray tube by the RO plane. Points $G_{2}$ and $G_{3}$ (actually, $G_{2}$ and $G_{3}$ are apparent radio images of transmitter $G_{1}$ as seen from points $B_{1}$ and $B_{2}$ ) are the intersections of the tangents
to the RO ray trajectory $G_{1} B_{1} B_{2} L$ at points $B_{1}$ and $B_{2}$ with the straight lines $O_{1} G_{1}$ and $O_{2} G_{2}$, respectively. The angles $\mu_{1}, \mu_{2}, \mu_{3}$, and $\mu_{4}$ have common vertices at points $G_{2}$ and $G_{3}$. Variables $\mu_{1}$ and $\mu_{2}$ are the angles between the tangent to the ray trajectory $G_{1} B_{1} B_{2} L$ at point $B_{1}$ and the directions of $O_{1} G_{2}$ and $O_{2} G_{2}$, respectively (Fig. 6). Variables $\mu_{3}$ and $\mu_{4}$ are the angles between the straight line $G_{3} B_{2}$ (the tangent to the ray trajectory $G_{1} B_{1} B_{2} L$ at point $B_{2}$ ) and the directions of $O_{2} G_{3}$ and $O_{3} G_{3}$, accordingly (Fig. 6). The dependence of the phase path excess and refractive attenuation on the impact parameter $p$ may be separately considered for three parts of the RO ray trajectory $G_{1} B_{1} B_{2} L$. The phase path $\Phi$ corresponding to the ray $G_{1} B_{1} B_{2} L$ (Fig. 6) is a sum

$$
\begin{array}{r}
\Phi=\sqrt{R_{1}^{2}-p_{1}^{2}}+\sqrt{R_{N}^{2}-p_{N}^{2}}+\sum_{i=1}^{i=N-1} b_{i} \cos \left(\gamma_{i}-\xi_{i}-\alpha_{i}\right) \\
+\sum_{m=1}^{N}\left[p_{m} \xi_{m}\left(p_{m}\right)+\kappa_{m}\left(p_{m}\right)\right] \tag{4}
\end{array}
$$

where $N$ is the number of spherical sectors influencing the trajectory $G_{1} B_{1} B_{2} L(N=3$ for the geometry shown in Fig. 6); $R_{i}(l-1)$ and $R_{i}(l)$ are the distances from the $i$ th spherical sector center $O_{i}$ to the input and output of the $l$ th spherical sector coinciding with boundaries $B_{l-1}$ and $B_{l}$; for the case $N=3, B_{0}$ and $B_{3}$ coincide with $G_{1}$ and $L$, respectively ( $B_{1}$ and $B_{2}$ are shown in Fig. 6); $R_{1}(0)=D_{1} ; p_{i}, \xi_{i}\left(p_{i}\right)$, and $\kappa_{i}\left(p_{i}\right)$ are the impact parameter, the bending angle, and the main refractivity part of the phase path corresponding to the $i$ th spherical sector, respectively. The refractive index at boundaries $B_{i-1}$ and $B_{i}$, with $i=2, \ldots, N$, is assumed to be equal to one. The distances $R_{i}(l-1)$ and $R_{i}(l)$ may be determined using the impact parameters $p_{i}$, with $i=1, \ldots, N$, if the angular coordinates of boundaries $B_{i-1}$ and $B_{i}$, defined by the angles $\Theta_{1}, \Theta_{2}$, and $\Theta_{3}$ (Fig. 6), are known.

An exact expression for the refractive attenuation $X_{L}$ is derived in the Appendix. Using the refractive attenuation in $N$
spherical sectors with coplanar centers, one can obtain

$$
\begin{align*}
X_{L}= & R_{0}^{2} \sin \gamma_{1} \\
& /\left\{R_{i} \cos \beta_{i}\left|\partial \theta_{i} / \partial \gamma_{1}\right| d_{2 N-2}(L) S(1) \cdots S(N)\right\} \\
& i=1, \ldots, N \\
d_{2 N-2}(L)= & R_{N}(L) \sin \left[\Theta_{N}(L)\right] \\
S(i)= & \sin \mu_{2 i-1} / \sin \mu_{2 i} \quad S(0)=1  \tag{5}\\
\theta_{i}(L)= & \pi+\xi\left(p_{1}\right)-\gamma_{i}-\beta_{i}, \quad i=1, \ldots, N  \tag{6}\\
\xi\left(p_{1}\right)= & \xi_{1}\left(p_{1}\right)+\cdots+\xi_{N}\left(p_{N}\right)  \tag{7}\\
p_{i}= & p_{i-1}+b_{i-1} \sin \left(\gamma_{i-1}-\xi_{1}-\cdots-\xi_{i-1}-\alpha_{i-1}\right) \\
& \quad i=2, \ldots, N . \tag{8}
\end{align*}
$$

The subscripts in (5)-(8) denote a number of spherical sectors, and the integer arguments correspond to a number of boundaries. $d_{2 N-2}(L)$ is the length of the perpendicular from point $L$ on the straight line linking the center of the $N$ th spherical sector (point $O_{N}$ ) with point $G_{N}$. The angles $\mu_{2 k-1}$ and $\mu_{2 k}$ have a common vertex at point $G_{k}$, and $\mu_{2 k-1}$ and $\mu_{2 k}$ are the angles between the ray tangent $G_{k} B_{k-1}$ and the straight lines $G_{k} O_{k-1}$ and $O_{k} G_{k}$, respectively. The relationship (5) for the refractive attenuation $X_{L}$ is valid for any arbitrary number $i$ from the interval $i=1, \ldots, N$.

The developed analytical model allows ray tracing of the RO signals. If the impact parameter $p_{1}$ and the refractive angle in the first sector $\xi\left(p_{1}\right)$ are known, then one can consequently determine the impact parameters $p_{i}$, and the refractive angles $\xi\left(p_{1}\right)$ with $i=2, \ldots, N$, from (7), (8), and then can recalculate the phase path $\Phi\left(p_{1}\right)$ and the refractive attenuation $X_{L}$ from (5), (6) for an arbitrary $N$. Note the important feature of the introduced model. If, in the $k$ th sector, the refraction effect is absent, then the dimension $N$ of the model can be lowered to $N-1$. In this case, the next equality fulfills

$$
\begin{equation*}
\mu_{2 k-1}=\mu_{2 k} \quad S(k)=1 \tag{9}
\end{equation*}
$$

In addition, (5) does not depend on the contribution from the $k$ th spherical sector.

The phase path of the electromagnetic waves after propagating through the ionosphere and atmosphere may be considered according to relationship (4) as a quasi-linear function of the refractive angles. Therefore, the linear methods of the ionospheric correction introduced earlier [12], [17] should be effective in the case of propagation through several spherical symmetric sectors and undisturbed ionosphere. However, the amplitude of the RO signals is a nonlinear function of the refractive angle, and in the case of the disturbed ionosphere, it is a subject for strong perturbations.

The ionospheric contribution can be significant at different altitudes of the RO ray perigee in the $40-90 \mathrm{~km}$ interval if the following two necessary and sufficient conditions are fulfilled: 1) the ionospheric part of the RO signal path contains a tangent point, and 2) there is a refractivity layer with a sharp gradient that is perpendicular to the ray $G_{1} B_{1} B_{2} L$ in the vicinity of the tangent point. In the simplest case, when an inclined plasma layer exists only on one part of the ray $G_{1} B_{1} B_{2} L$ and the influence of the neutral atmosphere is weak, the analytical
model predicts the displacement of the tangent point from the ray perigee $T$ to a plasma layer. As a result, one may observe unusually strong amplitude and phase variations of the RO signals in the 40-90 km interval of the perigee height $h(T)$. It follows that the height of the inclined ionospheric layers does not coincide with the altitude of the RO ray perigee. If the location of the tangent point and the center of the local spherical symmetry are known, then it is possible to restore the vertical gradient of refractivity using the Abel transformation. This extends the applicable domain of the RO method.

## IV. Identification of Plasma Layers

References [10], [11], and [15] detected and validated a connection between the phase excess $\Phi(p)$ (eikonal) acceleration and the refractive attenuation of the electromagnetic waves $X(t)$

$$
\begin{align*}
1-X(t) & =m a=m d F_{d} / d t
\end{align*}=m d^{2} \Phi(p) / d t^{2}, ~=q /\left(d p_{s} / d t\right)^{2} \quad q=\left(R_{0}-d_{2}\right) d_{2} / R_{0} . ~ l
$$

An inverse value $m^{-1}$ may be considered as a module of the tangent point $T$ centripetal acceleration oriented in the direction to point $L$. Formula (10) connects the refractive attenuation $X(t)$, the derivative of the Doppler frequency $F_{d}$ on time, and the phase acceleration $a=d F_{d}(t) / d t=d^{2} \Phi(p) / d t^{2}$ via a relationship that is similar to the classical dynamics equation. The variations of the refractive attenuation $1-X(t)$ may be considered as a ratio of the eikonal and centripetal accelerations of point $T$. The impact parameter $p_{s}$ and the distances $R_{0}$ and $d_{2}$ are shown in Fig. 1. Parameters $m$ and $d p_{s} / d t$ may be evaluated from the orbital data. The distance $d_{2}$ can be evaluated from the relationship [15]

$$
\begin{align*}
d_{2} & =2 R_{0} \beta\left[1+2 \beta(1-w / v)+(1-4 \beta w / v)^{1 / 2}\right]^{-1} \\
\beta & =m v^{2} / R_{0} \quad m=[1-X(t)] / a \tag{11}
\end{align*}
$$

where $w$ and $v$ are the velocity components of the GPS and LEO satellites, respectively, and they are perpendicular to the straight line $G L$ on the plane $G O L$ (Fig. 1). $w$ and $v$ are positive when they are oriented toward the point $O$ direction and negative in the opposite case. Equation (10) presents a possibility to convert the phase acceleration $a$ and/or Doppler frequency $F_{d}$ to the refractive attenuation $X_{p}$ [13], [15]. From these derived refractive attenuation and amplitude data, one can estimate the integral absorption of electromagnetic waves [15]. There is a possibility to apply the developed technique for the identification and location of the sharp layered sporadic Es structures in the ionosphere [13]. To consider the possibility to identify the plasma layer contribution in the RO signals, we will use a CHAMP RO event 0117 (January 14, 2001; 0 h 56 m LT; 76.4N, 172.7 W ) with strong quasi-regular amplitude and phase variations. The refractive attenuation of the CHAMP RO signals at the first GPS frequency $f_{1}$ (curve 1) and the phase path excesses at frequencies $f_{1}$ and $f_{2}$ (curves 2 and 3 ) as functions of the height of the RO ray perigee $h$ are shown in Fig. 7 (left panel). Curves 2 and 3 have been obtained after subtracting a regular


Fig. 7. Refractive attenuation at $f_{1}$ (curve 1) and the phase path excesses at $f_{1}$ and $f_{2}$ (curves 2 and 3 ; left). Comparison of the refractive attenuation at $f_{1}$ and the phase accelerations at $f_{1}$ and $f_{2}$ (curves 1,2 and 3 , right), respectively.
phase trend connected with the upper ionosphere influence. The form of the refractive attenuation variations indicates the impact of the ionospheric disturbances in the $72-98 \mathrm{~km}$ altitude. This disturbance consists of two patches which are responsible for the maxima in the intensity changes in the 72-78 and 8496 km intervals of $h$. In the 78-84 km interval of height $h$, the intensity variations are notable. However, they are not so strong. The phase changes at frequencies $f_{1}$ and $f_{2}$ in Fig. 7 (left panel) also indicate a two-layered structure at altitudes of 75 and 90 km . The phase accelerations at both frequencies $f_{1}$ and $f_{2}$ [curves 2 and 3 in Fig. 7 (right panel)] reveal the fine structures in the phase of the RO signals. The phase acceleration $a$ has been numerically estimated by double differentiation over a fixed time interval $\Delta t$. The value of $\Delta t$ was equal to 0.42 s . The strongest variations of the phase acceleration are observed almost in the same altitude intervals as that for the refractive attenuation. In this interval, the phase acceleration and refractive attenuation variations are strongly connected, and they may be considered as coherent oscillations caused by layered structures. It is important that, at altitudes of below 72 km and higher than 98 km , the refractive attenuation variations are small and that they do not have any connection with the changes of the phase acceleration [Fig. 7 (right panel)]. This indicates a different incoherent mechanism of the significant phase variations at the heights of $h \leq 72 \mathrm{~km}$ and $h \geq 98 \mathrm{~km}$. As a further identification step, a further examination is conducted to locate the indicated layers in the ionosphere. If parameter $m$ is estimated from the experimental data using (10), it is possible to find the new value of distance $T^{\prime} L \approx d_{2}^{\prime}$ and to determine the displacement of the new tangent point $T^{\prime}$ and the location of a layer relative to point $T$ (Fig. 1)

$$
\begin{equation*}
d=d_{2}^{\prime}-\left(R_{2}^{2}-p_{s}^{2}\right)^{1 / 2} \tag{12}
\end{equation*}
$$

The results of estimation of parameter $m$, displacement $d$, and corrected layer height $h^{\prime}$ are given in Table I as a function of the altitude $h$ of the ray perigee $T$. The data presented in Table I correspond to the CHAMP GPS RO event No. 0117.

TABLE I
Location of the Ionospheric Layers

| $h$ | $X_{p}-1$ | $X_{a}-1$ | $d, \mathrm{~km}$ | $h^{\prime}, \mathrm{km}$ | $\delta^{\circ}$ | $m, \mathrm{~s}^{2} / \mathrm{m}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 97.61 | 0.06429 | 0.07053 | 140.74 | 99.163 | 1.26 | 0.87587 |
| 97.58 | 0.06584 | 0.07291 | 155.00 | 99.461 | 1.38 | 0.88391 |
| 97.55 | 0.06694 | 0.07458 | 164.73 | 99.673 | 1.47 | 0.88940 |
| 97.52 | 0.06633 | 0.07550 | 198.30 | 100.59 | 1.77 | 0.90861 |
| 97.49 | 0.06610 | 0.07563 | 206.26 | 100.81 | 1.84 | 0.91316 |
| 97.46 | 0.06481 | 0.07478 | 219.46 | 101.22 | 1.96 | 0.92078 |
| 97.42 | 0.06299 | 0.07309 | 228.50 | 101.50 | 2.04 | 0.92599 |
| 97.39 | 0.06081 | 0.07047 | 226.61 | 101.41 | 2.02 | 0.92484 |
| 97.36 | 0.05814 | 0.06694 | 216.38 | 101.02 | 1.93 | 0.91883 |
| 97.33 | 0.05460 | 0.06265 | 210.95 | 100.81 | 1.88 | 0.91563 |
|  |  |  |  |  |  |  |
| 72.23 | 0.04132 | 0.06316 | 714.35 | 112.09 | 6.39 | 1.17436 |
| 72.19 | 0.04029 | 0.05866 | 626.54 | 102.86 | 5.60 | 1.11844 |
| 72.16 | 0.03934 | 0.05328 | 498.98 | 91.618 | 4.46 | 1.04047 |
| 72.13 | 0.03786 | 0.04721 | 357.26 | 82.105 | 3.19 | 0.95806 |

The variations of the refractive attenuations $X_{p}-1$ and $X_{a}-1$ calculated from the amplitude and phase data and the estimated $m$ values are shown in the second, third, and seventh columns in Table I as functions of the ray perigee height $H$ in the following two intervals: 1) $97.33-97.61 \mathrm{~km}$ and 2) $72.13-72.23 \mathrm{~km}$. These intervals correspond to a maximum in the absolute values of the refractive attenuation and phase acceleration variations that are relevant to curves 1 and 2, shown in Fig. 7 (right panel). The displacement $d$ and the estimated value of the layer altitude $h$ are presented in the forth and fifth columns in Table I, respectively. The displacement $d$ changes between $140-210$ and $357-714 \mathrm{~km}$ in the interval 1) and 2), respectively. These values correspond to the displacement of the tangent point from the ray perigee in the direction to point $G$. The corresponding corrections to the altitude $h$ are about $2-6$ and $20-30 \mathrm{~km}$ in the interval 1) and 2), respectively. This analysis suggests that the ionospheric layers are located in the entrance part of the ionosphere between point $G$ and $T$ (Fig. 1) at the distance $d$ in the interval of $300-500 \mathrm{~km}$. The corresponding values $\Delta h$ change in the $2-30-\mathrm{km}$ interval. The identification of the sporadic Es layer justifies the potential applications of the Abel transform in solving the inverse problem. The resulting


Fig. 8. Vertical distribution of the electron density and its gradient in the main parts of the two patches of the sporadic E-layer. Curves 1 and 3 describe the electron density distribution, and curves 2 and 4 are related to the vertical gradient of the electron density.
electron density distribution is shown in Fig. 8. As shown in Fig. 8, the electron density variations are concentrated in the interval $0<N(h)<3.5 \cdot 10^{10}$ [electrons $/ \mathrm{m}^{3}$ ]. These magnitudes of $N(h)$ are somewhat below the usual values of $N(h)$ for the sporadic E-layers. The height interval of the amplitude variations is nearly equal to the height interval of the variations in the electron density and its gradient. Two patches of the ionospheric layer are clearly shown in Fig. 8. The first patch of the layers is located on line $G T$ at a distance of 300 km from point $T$ (curves 1 and 2 in Fig. 8). It is concentrated in the 92-104-km interval, with an inclination to the horizontal direction $\delta$ of about $3^{\circ}$. The second patch (Fig. 8; curves 3 and 4) is located on the line $G T$ in the $94-100-\mathrm{km}$ interval at a distance of 500 km relative to the tangent point $T$ (Fig. 1), with an inclination of about $5^{\circ}$.

The plasma density is small between two patches. The amplitude variations in the $70-96-\mathrm{km}$ altitudes are associated with the sporadic E-layer located along the line $G T$ in $92-104 \mathrm{~km}$ at a distance of $300-500 \mathrm{~km}$ relative to point $T$. The inclination of the sporadic E-layer changes along the line $G T$ from $3^{\circ}$ to $5^{\circ}$. Therefore, the introduced method appears to have a considerable potential to resolve the uncertainty in the location of the inclined layer between the parts $G T$ and $L T$ of the ray trajectory. Additional validation of this method through the analysis of the CHAMP data and the comparison with the ground-based ionosonde information is the task of our future work.

## V. Conclusion

A new analytical model has been introduced to account for the local mechanism of the multiple-RO ionospheric effects, which incorporates the horizontal gradients in the ionosphere. The model gives analytical expressions for the phase path excess and the refractive attenuation of the electromagnetic waves propagating through the disturbed ionosphere. The new model can be used as a new analytical tool in calculating the parameters of the radio waves propagating through an inhomogeneous medium. The analytical presentation of the amplitude and phase variations of the radio waves gives some advantages compared with the phase screen method. The introduced analytical model suggests the potential for the applications of the Abel transform in estimating the electron density distribution
in the inclined ionospheric plasma layers. The analysis of the CHAMP RO data and the analytical model have demonstrated the importance of the comparative analysis of the amplitude and phase channels of the satellite radio-holograms in classifying the ionospheric influence on the RO signals. The preliminary analysis reveals the following five types of ionospheric impacts on the CHAMP RO signals at the altitudes of the RO perigee $40-90 \mathrm{~km}: 1)$ quiet events; 2) isolated quasi-regular flashes (possible contribution of the inclined sporadic E-layers); 3) events with quasi-periodical changes of the amplitude and phase (the possible source is the wave structures in the electron density); 4) diffractive events with a clearly identifiable diffraction pattern in the amplitude and phase; and 5) events with noisy contribution of the ionospheric disturbances to the amplitude. The noisy and quasi-regular amplitude variations in the RO signals correspond to the earlier described C- and S-type amplitude scintillations in the transionospheric satellite-to-Earth links. The CHAMP RO data analysis has shown the possibility of the identification, location, and evaluation of the electron density distribution and its gradient in the inclined ionospheric layers. The tangent point displacement is the main cause of the systematic error in the RO estimation of the altitude of the inclined ionospheric layers. This conclusion may also be valid in the case of electromagnetic wave propagation in the satellite-to-Earth communication links.

## Appendix <br> Derivation of the Refractive Attenuation

We choose a ray tube bounded by four surfaces. Two of them $\left(G_{1} F L C\right.$ and $\left.G_{1} A L^{\prime} D\right)$ are perpendicular to the plane $G_{1} B_{1} B_{2} L$, and they intersect this plane along the rays $G_{1} L$ and $G_{1} L^{\prime}$, respectively (Fig. 9). This ray tube has a nearly rectangular cross section $A F C D$ at point $L$, with the sizes $l_{\|}$ and $l_{\perp}$ in the plane surface $G_{1} B_{1} B_{2} L$ and in the perpendicular plane (Fig. 9). Owing to the local spherical symmetry, the tangents to the rays $G_{1} F, G_{1} L$, and $G_{1} C$ at points $B_{1}^{\prime}, B_{1}$, and $B_{1}^{\prime \prime}$ intersect the straight line $O_{1} G_{1}$ at point $G_{2}$ located in the plane $G_{1} B_{1} B_{2} L$ (Fig. 9). Therefore, $G_{2}$ is a radio image of transmitter $G_{1}$ as seen from the boundary of the first and second sectors (point $B_{1}$ ). The plane $O_{1} G_{1} B_{1} B_{2} L$ is the bisection of the surfaces $O_{1} G_{1} A F$ and $O_{1} G_{1} C D$. The surfaces $O_{1} G_{1} B_{1}^{\prime}$ and $O_{1} G_{1} B_{1}^{\prime \prime}$ form the two plane side walls of the ray tube in the first spherical sector. The planes $O_{1} G_{1} B_{1}^{\prime}$ and $O_{1} G_{1} B_{1}^{\prime \prime}$ intersect along the straight line $G_{1} O_{1}$, forming the dihedral angle $d \gamma$ (Fig. 9). The angle between the rays $G_{1} L$ and $G_{1} L^{\prime}$ in the RO plane $G_{1} B_{1} B_{2} L L^{\prime}$ at point $G_{1}$ is also assumed to be equal to $d \gamma$.

In the second spherical sector, these rays propagate according to the local spherical symmetry in the planes containing the center $O_{2}$ and the rays' tangents at points $B_{1}^{\prime}$ and $B_{1}^{\prime \prime}$. These planes are intersecting along the straight line $O_{2} G_{2}$, and they form the dihedral angle $d \gamma_{1}$. The distance $l_{\perp 1}$ at the boundary $B_{1}$ of the first spherical sector between the rays $G_{1} B_{1}^{\prime}$ and $G_{1} B_{1}^{\prime \prime}$ (Fig. 9) is

$$
\begin{equation*}
l_{\perp 1}=d_{1} d \gamma, \quad d_{1}=S_{1} \sin \mu_{1} \tag{1A}
\end{equation*}
$$



Fig. 9. Geometry of a ray tube of the electromagnetic waves propagating through three spherical symmetric sectors. Points $G_{1}, B_{1}, B_{2}$, and $L$ are supposed to be located in the same plane with the centers of the spherical symmetry $O_{1}, O_{2}$, and $O_{3}$. Points $G_{2}$ and $G_{3}$ are the intersections of the tangents to the ray trajectory $G_{1} B_{1} B_{2} L$ at points $B_{1}$ and $B_{2}$ with the straight lines $O_{1} G_{1}$ and $O_{2} G_{2}$, respectively. The angles $\Theta_{1}, \Theta_{2}$, and $\Theta_{3}$ have vertices at points $O_{1}, O_{2}$, and $O_{3}$. The values $\Theta_{1}, \Theta_{2}$, and $\Theta_{3}$ determine the angles between directions $O_{1} G_{1}, O_{2} G_{2}, O_{3} G_{3}$ and $O_{1} B_{1}, O_{2} B_{2}, O_{3} L$, correspondingly. The exit ray tube cross section $A F C D$ at point $L$ is nearly rectangular, and it has the sizes $l_{\|}$and $l_{\perp}$ in the plane surface $G_{1} B_{1} B_{2} L$ and in the perpendicular plane, respectively.
where $S_{1}$ is the distance $G_{2} B_{1}$. At the boundary $B_{1}$ in the second spherical sector, the distance $l_{\perp 2}$ between the rays $B_{1}^{\prime} B_{2}^{\prime}$ and $B_{1}^{\prime} B_{2}^{\prime \prime}$ is equal to

$$
\begin{equation*}
l_{\perp 2}=d \gamma_{1} d_{2}, \quad d_{2}=S_{1} \sin \mu_{2} \tag{2A}
\end{equation*}
$$

where $d_{1}$ and $d_{2}$ are the lengths of the perpendiculars from point $B_{1}$ on the straight lines $O_{1} G_{2}$ and $O_{2} G_{2}$, correspondingly. The size $l_{\perp}$ satisfies a continuity requirement at $B_{1}$

$$
\begin{equation*}
l_{\perp 1}=l_{\perp 2} \tag{3A}
\end{equation*}
$$

The next connection between the dihedral angles $d \gamma$ and $d \gamma_{1}$ follows from (1A)-(3A)

$$
\begin{align*}
& d \gamma_{1}=d \gamma d_{1} / d_{2}, \quad d_{1} / d_{2}=\sin \mu_{1} / \sin \mu_{2} \\
& d \gamma_{1}=d \gamma \sin \mu_{1} / \sin \mu_{2} \tag{4A}
\end{align*}
$$

From (2A) and (4A), the distance $l_{\perp 2}$ in the second sector can be expressed as

$$
\begin{equation*}
l_{\perp 2}=d \gamma_{1} d_{2}(E)=d \gamma d_{2}(E) \sin \mu_{1} / \sin \mu_{2} \tag{5A}
\end{equation*}
$$

where $d_{2}(E)$ is the length of the perpendicular from the current point $E$ on the ray $G_{1} L$ on the straight line $O_{2} G_{2}$. The same procedure can be applied in finding the dihedral angle $d \gamma_{2}$ at the boundary $B_{2}$ between the second and the third sectors

$$
\begin{align*}
d \gamma_{2} & =d \gamma_{1} d_{3} / d_{4}, \quad d_{3} / d_{4}=\sin \mu_{3} / \sin \mu_{4} \\
d \gamma_{2} & =d \gamma_{1} \sin \mu_{3} / \sin \mu_{4} \\
& =d \gamma \sin \mu_{1} \sin \mu_{3} /\left(\sin \mu_{2} \sin \mu_{4}\right) . \tag{6A}
\end{align*}
$$

The length $l_{\perp 3}$ of the ray tube at point $L$ can be found from the following equation:

$$
\begin{equation*}
l_{\perp 3}=d \gamma d_{4}(L) \sin \mu_{1} \sin \mu_{3} /\left(\sin \mu_{2} \sin \mu_{4}\right) \tag{7A}
\end{equation*}
$$

where $d_{4}(L)$ is the length of the perpendicular from point $L$ on the line $O_{3} G_{3}$ (Fig. 9). Owing to the local spherical symmetry, the tangents to the rays $G_{1} F, G_{1} L$, and $G_{1} C$ at points $B_{2}^{\prime}, B_{2}$, and $B_{2}^{\prime \prime}$ intersect the straight line $O_{1} G_{1}$ in the point $G_{3}$ located in the plane $G_{1} B_{1} B_{2} L$ (Fig. 9). $\mu_{3}$ and $\mu_{4}$ are the angles with a common vertex at point $G_{3}$ between the tangent to the ray $G F$ at point $B_{2}^{\prime}$ and directions $O_{2} G_{3}$ and $O_{3} G_{3}$, respectively (Fig. 9). From equations (2A)-(7A), the size $l_{\perp 3}$ then depends on the locations of the boundaries $B_{1}$ and $B_{2}$ and the centers of the spherical sectors $O_{1}, O_{2}$, and $O_{3}$.

The size $l_{\|}$at point $L$ may be found by considering the rays $G_{1} L$ and $G L^{\prime}$ in the plane $G_{1} L O_{1} O_{2} O_{3}$, which is the vertical section of the ray tube (Figs. 6 and 9). The central angles $\theta_{1}, \theta_{2}$, and $\theta_{3}$ between the directions to the transmitter $G_{1}$ and receiver $L$ have vertices in the centers $O_{1}, O_{2}$, and $O_{3}$ of the spherical sectors (Fig. 6). The impact parameters in the corresponding spherical sectors are designated by $p_{1}, p_{2}$, and $p_{3}$ (Fig. 6). Owing to the condition of spherical symmetry, the impact parameters $p_{1}, p_{2}$, and $p_{3}$ satisfy the following relationships, which are valid inside the $i$ th spherical sector:

$$
\begin{equation*}
p_{i}=n\left(R_{i}\right) R_{i} \sin \gamma_{e}, \quad i=1,2,3 \tag{8A}
\end{equation*}
$$

where $n\left(R_{i}\right)$ is the refractive index and $\gamma_{e}$ is the angle between the tangent to the ray trajectory $G_{1} L$ at the current point $E$ and direction to the center of the $i$ th spherical sector. The tangents to the ray trajectory $G_{1} L$ and the directions to the centers $O_{1}$, $O_{2}$, and $O_{3}$ make the angles $\gamma_{1}, \gamma_{2}$, and $\gamma_{3}$ at point $G_{1}$ and $\beta_{1}$, $\beta_{2}$, and $\beta_{3}$ at point $L$, respectively (Fig. 6). A method suggested in [14] is used to obtain an expression for $l_{\|}$. Three equivalent expressions are derived by considering the rectangular triangles having the common cathetus $L L^{\prime}\left(l_{\|}\right)$and the differentials $R_{i} d \theta_{i}$ as the hypotenuses (Fig. 6)

$$
\begin{equation*}
l_{\|}=R_{i}(L) \cos \beta_{i} \partial \theta_{i} / \partial \gamma_{1} d \gamma, \quad i=1,2,3 \tag{9A}
\end{equation*}
$$

where $R_{i}(L)$ is the distance $O_{i} L$ (Fig. 6). Note that the partial derivatives of the central angles $\theta_{i}$ that are relative to the input angle $\gamma_{1}$ are evaluated with the distances $R_{1}(L), R_{2}(L), R_{3}(L)$ and $D_{1}\left(G_{1}\right), D_{2}\left(G_{1}\right), D_{3}\left(G_{1}\right)$ held constant. The relationship (9A) is valid for an arbitrary number $i$, with $i=1,2,3$. The power $W$ emitted by an isotropic antenna in the ray tube is equal to

$$
\begin{equation*}
W=P d \Omega / 4 \pi=P \sin \gamma_{1}(d \gamma)^{2} / 4 \pi \tag{10~A}
\end{equation*}
$$

where $P$ is the power of the transmitter and $d \Omega$ is the solid angle that is relevant to the ray tube.

The power flow in free space $W_{L 0}$ is equal to

$$
\begin{equation*}
W_{L 0}=P / 4 \pi R_{0}^{2} \tag{11A}
\end{equation*}
$$

The refractive attenuation at point $L X_{L}$ can be obtained as a ratio of the power flow in the ray tube $W_{L}=W /\left(l_{\perp} \cdot l_{\|}\right)$to the
power flow in free space $W_{L 0}$ (11A)

$$
\begin{align*}
X_{L}= & W_{L} / W_{L 0} \\
= & R_{0}^{2} \sin \gamma_{1} /\left[d_{4}(L) R_{i}(L) \cos \beta_{i}\left|\partial \theta_{i} / \partial \gamma_{1}\right| \sin \mu_{1} \sin \mu_{3}\right. \\
& \left.\quad /\left(\sin \mu_{2} \sin \mu_{4}\right)\right] \\
& i=1,2,3 \quad d_{4}(L)=R_{3}(L) \sin \left[\Theta_{3}(L)\right] \tag{12~A}
\end{align*}
$$

where $\Theta_{3}(L)$ is the angle with the vertex at point $O_{3}$ between directions $O_{3} G_{3}$ and $O_{3} L$ (Fig. 9). The relationships between the impact parameters $\left(p_{1}, p_{2}\right.$, and $\left.p_{3}\right)$, central angles $\left(\theta_{1}, \theta_{2}\right.$, and $\theta_{3}$ ), and bending angles ( $\xi_{1}, \xi_{2}$, and $\xi_{3}$ ) can be obtained using the geometry of the path $G_{1} B_{1} B_{2} L$ (Fig. 6)

$$
\begin{align*}
p_{2} & =p_{1}+b_{1} \sin \left(\gamma_{1}-\xi_{1}-\alpha_{1}\right) \\
p_{3} & =p_{2}+b_{2} \sin \left(\gamma_{2}-\xi_{1}-\xi_{2}-\alpha_{2}\right)  \tag{13A}\\
\theta_{i} & =\pi+\xi\left(p_{1}\right)-\gamma_{i}-\beta_{i} \\
\xi\left(p_{1}\right) & =\xi_{1}\left(p_{1}\right)+\xi_{2}\left(p_{2}\right)+\xi_{3}\left(p_{3}\right), \quad i=1,2,3 \tag{14A}
\end{align*}
$$

Relationships (4) and (12A)-(14A) present the main content of the analytical model in the partial case of three spherical sectors. In plane $G_{1} B_{1} B_{2} L$, the phase path and the refractive attenuation depend on the sum of the phase changes and the bending angles in the spherical sectors, respectively, and they practically do not depend on the location of their boundaries. The effects of a spherical symmetric layer do not significantly depend on its location in the first, second, or third spherical sectors.

The introduced model may be generalized for an arbitrary number $N$ (i.e., the number of spherical symmetric sectors with the centers of the spherical symmetry located in the same plane).

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